MATHEMATICS - Sample questions

Question 1 – General Mathematics (+)

The greatest value of the function $f(x) = -\frac{x^4}{4} + \frac{x^3}{3} + 3x^2 - 15$ over $\mathbb{R}$ is:

(a) $\frac{-29}{3}$

(b) $-\frac{1}{4}$

(c) -15

(d) $\frac{3}{4}$

(e) $\frac{17}{12}$

Question 2 – General Mathematics (+)

Let $A = \int_0^1 (x - 3x^2)\,\mathrm{d}x + \int_0^{\frac{\pi}{3}} (\cos x + \sin x)\,\mathrm{d}x$. Then $A =$

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{\sqrt{3}}{2} - 1$

(c) $\frac{1}{2}$

(d) $1 + \frac{\sqrt{2}}{2}$

(e) $-\frac{1}{2} - \frac{\pi}{3}$

Question 3 – General Mathematics (+)

Let z and z' be two complex number such that $|z| = |z'| = 1$, and $\arg(z) = \theta$, $\arg(z') = \theta'$. Consider

$$Z = \frac{z + z'}{1 + zz'}$$

then

(a) $Z = \dfrac{\sin(\theta - \theta')}{\sin(\theta + \theta')}$

(b) $Z = \dfrac{\cos(\theta + \theta')}{\cos(\theta - \theta')}$

(c) $Z = \dfrac{\cos((\theta - \theta')/2)}{\cos((\theta + \theta')/2)}$

(d) $Z = \dfrac{\cos((\theta + \theta')/2)}{\cos((\theta - \theta')/2)}$

(e) $Z = \dfrac{\sin((\theta + \theta')/2)}{\sin((\theta - \theta')/2)}$

Question 4 – General Mathematics (+)

Let $f(x) = \sqrt{x^2 + x} + 3x - 1$. Then, as $x \to +\infty$, $f(x) =$

(a) $3x - 1 + o(1)$

(b) $3x + 2 + o(1)$

(c) $4x - 1 + o(1)$

(d) $4x - \frac{1}{2} + o(1)$

(e) $4x + o(1)$

Question 5 – General Mathematics (++)

Let $(u_n)_{n\in\mathbb{N}}$ be a sequence of real numbers. Which of these assertion is equivalent to "u_n does not converges toward 0".

(a) $\exists N \in \mathbb{N}, \forall \varepsilon > 0, \forall n \geq N, |u_n| \leq \varepsilon$

(b) $\forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n \geq N, |u_n| \leq \varepsilon$

(c) $\forall N \in \mathbb{N}, \forall \varepsilon > 0, \forall n \geq N, |u_n| \geq \varepsilon$

(d) $\exists N \in \mathbb{N}, \exists \varepsilon > 0, \forall n \geq N, |u_n| \geq \varepsilon$

(e) $\exists \varepsilon > 0, \forall N \in \mathbb{N}, \exists n \geq N, |u_n| \geq \varepsilon$

Question 6 – General Mathematics (++)

Let $\mathcal{C}$ be the graph of the function

$$f(x) = x^2 - 1$$

and let O denote the point $(0, 0)$.

Then $\inf_{M\in\mathcal{C}} OM =$

(a) $\sqrt{2}$

(b) 1

(c) $\sqrt{3}/2$

(d) $\sqrt{5/8}$

(e) $1/\sqrt{2}$

Question 7 – Analysis (+)

The value of the integral

$$I = \int_0^1 \frac{\mathrm{d}x}{1 + e^x}$$

is

(a) $\ln(2)$

(b) $1 - \ln(1 + e) + \ln(2)$

(c) $\ln(e) + \ln(2)$

(d) $1 - \ln(1 + e)$

(e) $\ln(1 + e)$

Question 8 – Analysis (+)

Let y denote the solution of

$$\begin{cases} y'' + y & = 0 \\ y(0) = 2, y'(0) & = -1 \end{cases}$$

where $y' = \mathrm{d}y/\mathrm{d}x$, $y'' = \mathrm{d}^2y/\mathrm{d}x^2$. Then $y(\pi/2) =$

(a) -1

(b) 0

(c) 1

(d) 2

(e) there is no such y.

Question 9 – Analysis (++)

What is the value of the quantity

$$\sum_{k=0}^{n} \binom{n}{k} \frac{1}{k+1}$$

Hint: you can first evaluate the integral $\int_0^1 x^k \mathrm{d}x$.

(a) 2^n

(b) $\dfrac{2^{n+1}-1}{n+1}$

(c) $\dfrac{2^n-1}{n+1}$

(d) $\dfrac{2^n}{n}$

(e) none of the above

Question 10 – Algebra (+)

For $n \geq 1$, we consider the polynome $P_n(X) := \sum_{k=0}^{n-1} X^{2k}$. What are the roots of P_n?

(a) $\{e^{i\frac{2\pi k}{n}}, k = 1, ..., 2n-2\}$

(b) $\{e^{i\frac{2\pi k}{n}}, k = 1, ..., n-1\}$
$\bigcup \{e^{-i\frac{2\pi k}{n}}, k = 1, ..., n-1\}$

(c) $\{e^{i\frac{\pi k}{n}}, k = 1, ..., 2n-2\}$

(d) $\{e^{i\frac{\pi k}{n}}, k = 1, ..., n-1\}$

(e) $\{e^{i\frac{\pi k}{n}}, k = 1, ..., n-1\}$
$\bigcup \{e^{-i\frac{\pi k}{n}}, k = 1, ..., n-1\}$

Question 11 – Algebra (++)

The matrix

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & 2 & -3 \\ -2 & 2 & 1 \end{pmatrix}$$

is

(a) invertible and diagonalizable on $\mathbb{R}$

(b) invertible but not diagonalizable on $\mathbb{R}$

(c) not invertible and not diagonalizable on $\mathbb{R}$

(d) not invertible but diagonalizable on $\mathbb{R}$

(e) not invertible and not triangularizable on $\mathbb{R}$

Question 12 – Algebra (++)

Let the sequence $(u_n)_{n\geq 0}$ of real numbers be defined by the relations $u_0 = 2$, $u_1 = 3$, and

$$u_{n+2} = \frac{1}{6}u_{n+1} + \frac{1}{6}u_n \quad \forall n \geq 0.$$

Then $\lim_{n\to+\infty} u_n =$

(a) $\frac{13}{5}$

(b) 0

(c) $\frac{2}{3}$

(d) $+\infty$

(e) the limit does not exist

Question 13 – Probability (+)

An urn contains 8 white balls and 4 black balls. We successively draw 3 balls without replacement. What is the probability for the first drawn ball to be black knowing that at least one black ball was obtained after the three draws?

(a) $\dfrac{55}{123}$

(b) $\dfrac{1}{2}$

(c) $\dfrac{3}{8}$

(d) $\dfrac{31}{121}$

(e) none of the above

Question 14 – Probability (++)

We consider two positive random variables X and Y of expectation 1. Further we assume that the variance of X is 3 and the variance of Y is 2.

(a) $\mathbb{E}[X+Y] = 2$ and $var(X-Y) = 1$

(b) $\mathbb{E}[3X] = 3$ and $var(3X) = 9$

(c) $\mathbb{E}(X \geq 4) \leq 1/4$ and $\mathbb{P}(Y \geq 4) \leq 2/9$

(d) $\mathbb{P}(X \geq 1) \geq 1/2$ and $\mathbb{P}(X \geq 2) \leq 1/2$

(e) $\mathbb{E}(X-Y) = 0$ and $\mathbb{P}(X = Y) = 0$

Question 15 – Probability (++)

A test to detect a disease is positive with probability 0.9 when the disease is present and negative with probability 0.99 when the disease is absent. 1% of the population is affected by the disease. What is the probability that an individual positive to the test is affected by the disease?

(a) 0.524

(b) 0.9

(c) 0.99

(d) 0.91

(e) 0.853

PHYSICS - Sample questions

Question 1 – Mechanics (+)

Two astronauts are in orbit, far away from the earth and from any other planets or satellites, therefore there is no gravitational force acting on them. They are at rest, close to each other. Suddenly, the astronaut A, which have a mass m_A, pushes the astronaut B, which has a mass m_B. After that action, the astronauts B start moving with a speed vector $\vec{v}_B$. What will be the speed of the astronaut A?

(a) $\vec{v}_A = \frac{m_A}{m_B}\vec{v}_B$

(b) $\vec{v}_A = -\frac{m_A}{m_B}\vec{v}_B$

(c) $\vec{v}_A = -\vec{v}_B$

(d) $\vec{v}_A = -\frac{m_B}{m_A}\vec{v}_B$

(e) $\vec{v}_A = 0$

Question 2 – Mechanics (++)

A solid sphere of mass m and radius r rolls down an inclined plane, without sliding. If the sphere starts from rest, what is its speed after its vertical height has decreased by an amount h? The momentum of inertia for a solid sphere is $I = 2\,m\,r^2/5$.

(a) $\sqrt{gh}$

(b) $\sqrt{2gh}$

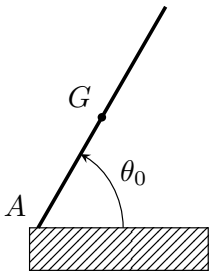
(c) $\sqrt{5gh}$

(d) $\sqrt{\frac{10}{7}gh}$

(e) $\sqrt{\frac{5}{2}gh}$

Question 3 – Mechanics (+++)

A bar of length 2ℓ, mass m and moment of inertia $J = \frac{1}{3}m\ell^2$, is fixed to the ground at a point A (see figure). At time $t = 0$, the bar makes an angle θ_0 with respect to the ground. The initial speed of the center of mass of the bar is zero. We will suppose that the friction is so that the point A is fixed while the bar moves. What is the speed of the center of mass when the bar touches the ground ?



(a) $v_G = \sqrt{\frac{3g\ell}{5}\sin\theta_0}$

(b) $v_G = \sqrt{\frac{3g}{2\ell}\sin\theta_0}$

(c) $v_G = \sqrt{\frac{2g\ell}{3}\sin\theta_0}$

(d) $v_G = \sqrt{\frac{3g\ell}{2}\cos\theta_0}$

(e) $v_G = \sqrt{\frac{3g\ell}{2}\sin\theta_0}$

Question 4 – Optics (+)

An object (size 4 cm) is placed at 10 cm from a thin converging lens of focal 5 cm. An image of this object is formed on a screen. What is the correct observation?

(a) The image is at 10 cm, with a size of 4 cm

(b) The image is virtual, at the side of the object

(c) The image is at 20 cm, with a size of 2 cm

(d) The image is at 20 cm, with a size of -2 cm

(e) The image is at 10 cm, with a size of -4 cm

Question 5 – Optics (++)

A Michelson interferometer adjusted in constructive interferences is illuminated with a white source. One inserts in the fixed arm a transparent plate of thickness $e = 100$ µm and index $n = 1.3$. Which displacement (d) of the moving mirror is necessary to retrieve these constructive interferences.

(a) $d = 10$ cm

(b) $d = 15$ mm

(c) $d = 30$ µm

(d) $d = 1$ µm

(e) $d = 3$ mm

Question 6 – Electromagnetism/Waves (+)

Two identical electrostatic charges ($Q = 10^{-10}$ C) with the same mass (M) are placed in vacuum at a distance of 1 m. What is their masses in order to have the same gravitation force as the electrostatic one?

(a) 4.1×10^5 kg

(b) 2.6×10^3 kg

(c) 1.1 kg

(d) 4.7×10^2 kg

(e) 10 kg

Question 7 – Electromagnetism/Waves (+)

Let us consider a cylinder of infinite length (along the axis y), and with cross section of size a (along the axis x and z). The walls of the cylinder are perfectly conductive, whereas the inside of the cylinder is empty and free of charge. Which equation does satisfy the electric potential V ?

(a) $V = 0$

(b) $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = 0$

(c) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

(d) $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = 0$

(e) $\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

Question 8 – Electromagnetism/Waves (++)

In a plane wave model (wave vector $\mathbf{k}$, pulsation ω), the complex representation of the magnetic field strength $\mathbf{H}$(A/m) associated to an electromagnetic wave in vacuum writes $\mathbf{H} = \mathbf{H_m} \exp(i(\mathbf{k}.\mathbf{r} - \omega t)$. The magnitude $|\mathbf{E_m}|$ of the electric field then writes:

(a) $|\mathbf{E_m}| = \frac{\mu_0}{c}|\mathbf{H_m}|$

(b) $|\mathbf{E_m}| = \mu_0 c|\mathbf{H_m}|$

(c) $|\mathbf{E_m}| = \varepsilon_0 \mu_0 c|\mathbf{H_m}|$

(d) $|\mathbf{E_m}| = c|\mathbf{H_m}|$

(e) $|\mathbf{E_m}| = \frac{c}{\mu_0}|\mathbf{H_m}|$

Question 9 – Thermodynamics (+)

Evaluate the mean distance between two particles in the air under normal conditions ($P = 1$ atm et $T = 273$ K).

(a) 3.3 nm

(b) 0.3 µm

(c) 0.4 nm

(d) 12.4 µm

(e) 53 nm

Question 10 – Thermodynamics (++)

An air conditionner works in ideal condition to cool and to keep an amphitheater at temperature $T_i = 300$ K. The external temperature is $T_e = 330$ K. Suddenly, the lights of the amphitheater, whose total power is $P = 1000$ W, are switched on. Which increment of power ΔW the air conditionner must supply to keep the internal temperature of the amphitheater ?

(a) 66 W

(b) 100 W

(c) 10 W

(d) 1000 W

(e) 30 W

Question 11 – Quantum mechanics (+)

In an infinite potential well of width a, the energy of the n-th quantum state is given by $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$. If the particle in this quantum well is an electron, $\frac{\pi^2\hbar^2}{2m} \approx 400$ meV nm^2. For $a = 20$ nm, what is the energy difference between the ground state and the first excited state of this electron?

(a) 1 meV

(b) 3 meV

(c) 4 meV

(d) 8 meV

(e) 9 meV

Question 12 – Quantum mechanics (++)

A hypothetical quantum system is described by a wave function of the form $\varphi(x) = A(1 - x^2)$ if $x \in (-1, 1)$ and $\varphi(x) = 0$ outside this interval. What is the value of A?

(a) $15/16$

(b) $16/15$

(c) $4/\sqrt{15}$

(d) $\sqrt{15}/4$

(e) 1

CHEMISTRY - Sample questions

Question 1 – General chemistry (+)

In the case of this endothermic dissolution process
Ag_2CrO_4 (s) = 2 Ag^+ (aq) + CrO_4^{2-} (aq)
which of these actions will shift the balance to the right?

(a) Addition of NaCl, which causes the precipitation of AgCl

(b) Pressure increase

(c) Addition of $AgNO_3$, soluble in water

(d) Addition of an excess of Ag_2CrO_4 (s)

(e) Temperature decrease

Question 2 – General chemistry (++)

A triacid of a H_3A type (with a molecular mass of M = 192 g. mol-1) has the three following pKa:
pKa1 = 3.14, pKa2 = 4.77, pKa3 = 6.39
21.4 g of sodium dihydrogenosel (NaH_2A) plus 11.8 g of sodium monohydrogenosel (Na_2HA) are dissolved in 1L of pure water (we will neglect volume variations here in order to always have a solution of 1L).
What is the pH of the solution obtained?
M(Na) = 23 g.mol^{-1}; M(H) = 1 g.mol^{-1} ;
$\log(\frac{1}{2}) = -0,30$

(a) 3.14

(b) 3.96

(c) 4.47

(d) 4.77

(e) 5.58

Question 3 – General chemistry (+++)

Acetylene (ethyne C_2H_2) is a gas that provides a very bright flame during combustion. It can be produced from the following reaction:
CaC_2 (s) + $2H_2O$ (l) = $Ca(OH)_2$ (s) + C_2H_2 (g)
100 g of calcium carbide (CaC_2) react with 100 mL of water (H_2O). Considering that standard conditions, at 298 K, are respected, what is the exhaust volume of ethyne?

(a) 20 L

(b) 25 L

(c) 30 L

(d) 35 L

(e) 50 L

Data
ρ (water) = 1 kg.L^{-1}, M (Ca) = 40 g.mol^{-1}, M (C) = 12 g.mol^{-1}, M (O) = 16 g.mol^{-1}, M (H) = 1 g.mol^{-1}, R = 8.314 J.mol^{-1}.K^{-1}.

Question 4 – Chemical thermodynamics (++)

Considering the overall reaction:

2 SO_2(g) + O_2 (g) = 2 SO_3(g)

If an equimolar mixture of SO_2 and O_2 is introduced in a chemical reactor, assuming the reaction is total, the limiting reagent is:

(a) O_2

(b) SO_3

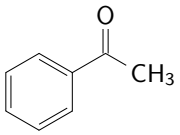
(c) There is no limiting reagent

(d) Both SO_2 and O_2 can be considered as limiting reagent

(e) SO_2

Question 5 – Organic chemistry (+)

The following compound is a/an:



(a) aldehyde

(b) ketone

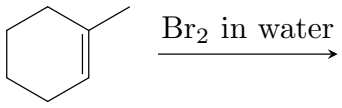
(c) ether

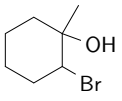
(d) ester

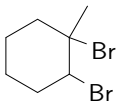
(e) carboxylic acid

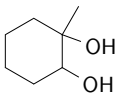
Question 6 – Organic chemistry (++)

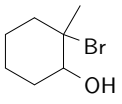
What is the main compound A obtained by the following reaction ?

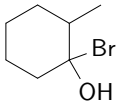


(a) 

(b) 

(c) 

(d) 

(e) 

LIFE SCIENCES - Sample questions

Question 1 – Life Sciences (+)

Plants are regularly subjected to biotic attacks by bacteria, fungi, or viruses. Three plant hormones may be involved in intercellular signaling to activate defense responses in nearby cell sites of infection:

(a) folic acid, defensins and polygalacturonases

(b) progesterone, estradiol and testosterone

(c) auxin, cytokinins and strigolactones

(d) salicylic acid, ethylene and jasmonic acid

(e) creatine, somatotropin and somatoliberin

Question 2 – Life Sciences (++)

RubisCO (Ribulose-1,5-diphosphate Carboxylase / Oxygenase) is a plant enzyme involved in the Calvin cycle and more particularly in the fixation of carbon dioxide. This Calvin cycle:

(a) is an overall oxidation of pentoses

(b) strictly works in the light

(c) reduces both the carbon and oxygen assimilated

(d) works in a synchronized manner with the Klein cycle

(e) requires ATP and NADPH to cover the energy cost of reducing mineral carbon

Question 3 – Life sciences (++)

A so-called "orthodox" seed is:

(a) an healthy seed, devoid of any pathogen

(b) a seed displaying no dormancy

(c) a seed with extremly low water content and in a quiescent metabolic state

(d) a seed without embryo

(e) a seed without endosperm

Question 4 – Life sciences (+++)

Paul crossed an homozygous plant with red flowers with an homozygous plant with white flowers. He got a plant with red flowers. He then self-fertilized this plant and among the descendants of it, he counted 298 plants with red flowers and 102 plants with white flowers. That means:

(a) selfing failed

(b) the color of the flowers is determined by 1 gene with 2 alleles, the red allele being codominant with the white allele

(c) the color of the flowers is determined by 1 gene with 2 alleles, the red allele being dominant on the white allele

(d) the color of the flowers is determined by 2 genes, the first one coding for the red allels, the second one for the white allels

(e) the starting plants were not homozygous